Determining the General Gas Law Constant, *R*

OVERVIEW

Recently in my physics class my imagination caught fire. My teacher derived the gas law equation $PV = \frac{1}{3}Nm\overline{c^2}$ using classical mechanics: equations of motion, impulse, etc., that is, using pure theory alone. My mind was ready for further exploration, and so when we were told about an IA for physics I once again turned my attention to the gas laws, specifically the universal gas constant, denoted "*R*".

The universal gas law constant is important because it is applicable to all gases in the ideal gas law, and it is also applied in various equations in the scientific field in different forms. That such a law is possible has fascinated me since I first learned about it in physics class.

The purpose of my investigation, then, is to experimentally determine the value of the universal gas constant. First, the experimental value of Boyle's law constant was determined—the product of pressure P and volume V for a constant temperature T and a fixed mass m of air. Then, I measured the mass of air indirectly and looked up the molar mass of air. Next I determined the number of moles in my experiment's air sample. With these values, the universal gas law constant R was calculated from the equation $PV = nRT$.

My experimental value for R was 8.05 J mol^{-1} K^{-1} whereas the accepted[1] or textbook value is 8.3144621 J mol^{-1} K^{-1}. Comparing the two values my result is only off by about 3%. However, accounting for experimental uncertainties (in the worst case) by results are good to 8%. This is still somewhat acceptable.

Comment [A1]: Personal Engagement. As we will see, the student is excited, even inspired, to conduct an IA concerning the general gas law.

Comment [A2]: Personal Engagement. The student is clearly interested in this, and now they will measure the constant experimentally.

Comment [A3]: Exploration. This is a standard high school experiment, but the student puts much thought and critical skills into the experiment. The method is nicely stated, and is ambitious to say the least.

BACKGROUND

Comment [A4]: Exploration. The scientific context is nicely described.

In the 17th century Robert Boyle investigated the relationship between the pressure and volume of a gas; he discovered that for a fixed temperature and constant mass, the product of pressure and volume were constant. In the early 1800's the study of gas became important, as the use of hot air balloons was popular; scientists wanted to improve the efficiency of hot air balloons. Jacques Charles investigated the effects of temperature on a gas, and Joseph Louis Gay-Lussac formulated what is now known as Charles's law: namely, that volume was proportional to temperature for a fixed mass and constant pressure.[2,3] Boyle's and Charles' results can be combined with a proportionally constant r unique to each type of gas, and it was written as $PV = rT$.

This equation requires a different proportional constant r for each specific gas. And yet scientists were not satisfied with this equation. What they wanted was one constant that could be used for every type of gas. Then, with Avogadro's discovery that at a given temperature and pressure, that equal volumes of any gas contain an equal number of molecules, the general gas law was formulated. By carefully investigating the relationship between the gas constant and the molar mass of a gas, scientists realized that there was a constant ratio between them, which is the **universal gas constant**, denoted ***R***. The general gas law is written as $PV = nRT$. Although this constant is for an ideal gas, it is approximately appropriate for real gases given the limit range of pressure, temperature and volume values.

Comment [A5]: Exploration. The student has demonstrated a thorough understanding of their topic.

The general gas law constant *R* is one of the most important physical quantities in the natural sciences. It describes the relationship of individual gas constants and molar mass. It appears in various aspects of science, including the ideal gas law in thermo-physics and the Nernst equation in electrochemistry[4]. Moreover, ideologically the universal gas constant is equivalent to the Boltzmann constant[5], which helps define entropy statistically and has an important role in science.

Comment [A6]: Exploration. The student offer more insightful background and context to the universal gas constant.

INTRODUCTION

Comment [A7]: Communication. Sections are easy to understand, and the logical flow makes good sense. The student offers a number of insights as well as a sound scientific study.

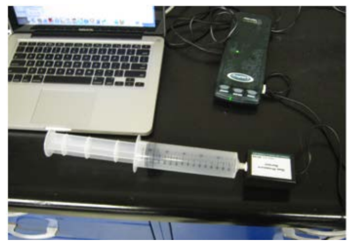
The ideal gas law equation[6] of pressure *P*, volume *V*, the number of moles *n* and the temperature *T* will be used to experimentally determine the universal gas constant *R*.

First I establish the proportionally constant of Boyle's law ($P\,V$ = constant) for a given sample of air at room temperature. Then I establish the molar volume of the sample and hence calculate *R* as follows:

$$PV = \text{constant} \quad \rightarrow \quad PV = nRT \quad \rightarrow \quad R = \frac{\text{constant}}{nT}$$

EXPERIMENTAL MATERIALS

Mercury Thermometer	Data-Logger Interrface, Vernier's *LabPro*	140 cc Monoject Plastic Syringe
		

Pressure Sensor, Vernier GPS-BTA	Data Processing Software, Vernier's *LoggerPro* V3.8.4 for Mac
	

MEASUREMENTS & UNCERTAINTIES

Comment [A8]: Exploration. Here and in other places the student is fully aware of errors, uncertainties and assumptions.

Volume. The syringe is calibrated in cubic centimeters with markings every 5 cc. It is safe to estimate the measurement to 1 cc; hence the uncertainty will be ±1 cc. The uncertainty ranges from 0.7% at the largest volume up to just under 3% for the smallest volume.

$$\Delta V_n\% = \frac{1\text{cc}}{V_n} \times 100$$

The percentage for each proved to be so small so I decided to ignore them and not plot them on my graphs.

Pressure. Pressure was measured with *Vernier's* GPS-BTA[6] pressure sensor. This sensor measures absolute pressure and the manufacture claims it needs no calibration. The gas sensor specifications[7] also states that the sensor has a resolution of 0.05 kPa. This is the uncertainty in the measured absolute or raw data values of pressure, $\Delta P = \pm 0.05\,\text{kPa}$.

My experiment utilizes the average of fives sets of pressure measurements. I then determined the average pressure. The uncertainty used for each averaged pressure value is ± one-half the range, although this is the uncertainty for the raw data and not a statistical uncertainty for the average pressure.

Comment [A9]: Exploration. Five repeated measurements for any given volume is more than adequate, and the range is acceptable given the equipment available in the high school lab.

This absolute uncertainty varied from ±0.2% for the largest volume up to just over ±2% for the smallest volume. The error bars for this would be even smaller than the error bars for the volume, which we excluded from my graph because they could not be seen. However, this percentage of uncertainty in the range is greater than the uncertainty in the average of the range. For the uncertainty in the averages I found the statistical value of standard deviation, and this was much smaller than 2%. In fact, for the smallest volume of 35 cc (with the greatest range uncertainty of 2%) the standard deviation is $\sigma = 1.7$, and this is a statistical uncertainty of only ±0.8%. For the largest volume, the standard deviation in the pressure values was only 0.2 kPa or about one-third of one-percent of the measured pressure value. These calculations were made using a web site[8].

Because the uncertainty is so small it is safe to assume that the uncertainty in the reciprocal of pressure is the same percentage, even though the reciprocal function is non-symmetrical. When you consider significant figures the asymmetry of the reciprocal functions disappears when the percentage is small. Hence I feel it is safe to not include error bars in Graph 2.

Room Temperature. The room temperature was measured and recorded just before I performed the experiment. The work was in the physics lab basement. The altitude here is approximately 1958 m above sea level. The thermometer read 21.0 °C with an estimated uncertainty of ±0.1C°. This is less than 0.5%. I made no effort at calibrating values from the thermometer and barometer due the possible change in mercury density (as I was told that would be a very minor change).

EXPERIMENTL PROCESURE

Comment [A10]: **Exploration**. The method is most appropriate. The student clearly knows what they are doing.

After measuring and recording the room temperature, I sealed the syringe with 140 cc of air in it to the pressure sensor. This was connected to the *LabPro* data processing unit by *Vernier* and then to a computer interfacing with *Vernier's LoggerPro* software.

I compressed the syringe in steps of 5 cc and recorded the pressure. This was done over a range from 140 cc down to 35 cc. At volumes from around 60 cc to 35 cc the pressure was extreme, and so I had a friend help hold the syringe as I recorded the data.

After recording the volume and pressure measurements over the given range I then repeated the experiment for a total of five times. I used the average pressure for a given volume when making calculations.

DATA

Comment [A11]: **Analysis**. The student has selected, recorded and processed the data in a beautifully executed way. They have also interpreted the data in a most relevant and appropriate way.

The spreadsheet function of *Logger Pro* was programmed to perform the obvious calculations of averaging the five sets of data, taking the reciprocal of the average pressure data, finding the product of pressure and volume, finding the uncertainty in the volume, converting units, etc. The complete data table in *Logger Pro* was too big to show all these detail, but I have included in the data table the important raw data and averaged data and a few other calculations.

The absolute uncertainty in the volume was $\Delta V = \pm 1\text{cc}$ and the absolute uncertainty in the raw pressure data was $\Delta P = \pm 0.05\text{ kPa}$, and the other uncertainties are discussed in this report. Graph 2 uses SI units of meter-squared for volume and Pascals for pressure, and in my calculations of the number of moles will use mass in grams.

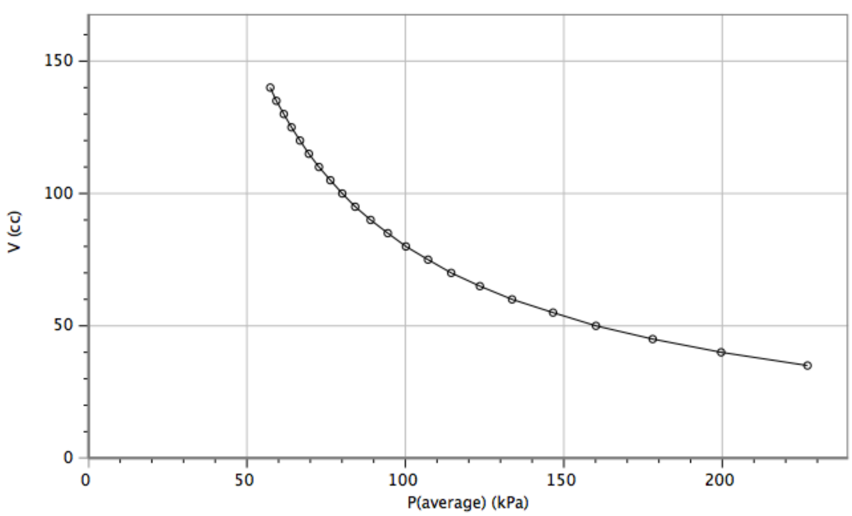
	Data Set									
	Volume (cc)	Pressure Set 1 (kPa)	Pressure Set 2 (kPa)	Pressure Set 3 (kPa)	Pressure Set 4 (kPa)	Pressure Set 5 (kPa)	Average Pressure (kPa)	1/P(average) (kPa^{-1})	P V (kPa * cc)	V Uncertainty (%)
1	140.0	57.59	57.17	57.24	57.25	57.58	57.366	0.017432	8031.240	0.71
2	135.0	59.50	59.12	59.22	59.24	59.4	59.296	0.016865	8004.960	0.74
3	130.0	61.80	61.39	61.57	61.76	61.74	61.652	0.016220	8014.760	0.77
4	125.0	64.08	63.85	64.01	64.36	64.05	64.070	0.015608	8008.750	0.80
5	120.0	66.83	66.61	66.73	66.83	66.75	66.750	0.014981	8010.000	0.83
6	115.0	69.46	69.46	69.57	69.64	69.57	69.540	0.014380	7997.100	0.87
7	110.0	72.67	72.3	72.72	72.98	73.04	72.742	0.013747	8001.620	0.91
8	105.0	76.66	75.98	76.16	76.62	76.33	76.350	0.013098	8016.750	0.95
9	100.0	79.94	79.97	80.05	80.19	80.31	80.092	0.012486	8009.200	1.00
10	95.0	84.26	83.95	84.1	84.15	84.4	84.172	0.011880	7996.340	1.05
11	90.0	89.33	88.93	88.76	88.87	89.31	89.040	0.011231	8013.600	1.11
12	85.0	94.52	95.01	93.73	94.36	94.72	94.468	0.010586	8029.780	1.18
13	80.0	100.16	99.98	99.68	100.41	100.59	100.164	0.009984	8013.120	1.25
14	75.0	107.08	107.02	106.83	107.35	107.52	107.160	0.009332	8037.000	1.33
15	70.0	114.54	114.34	114.26	114.99	114.17	114.460	0.008737	8012.200	1.43
16	65.0	123.32	123.45	123.34	123.75	123.69	123.510	0.008097	8028.150	1.54
17	60.0	134.06	133.59	132.82	134.66	133.2	133.666	0.007481	8019.960	1.67
18	55.0	146.44	147.36	144.85	147.33	147.02	146.600	0.006821	8063.000	1.82
19	50.0	160.83	160.87	158.13	160.94	159.91	160.136	0.006245	8006.800	2.00
20	45.0	177.98	178.02	176.72	179.7	177.77	178.038	0.005617	8011.710	2.22
21	40.0	200.70	200.29	197.18	199.38	200.62	199.634	0.005009	7985.360	2.50
22	35.0	226.27	226.89	224.9	229.63	226.77	226.892	0.004407	7941.220	2.86

BOYLE'S LAW CONSTANT FOR AIR

The first analysis is to determine the Boyle's law constant for the given mass of air at room temperature sealed in the syringe. Here in Graph 1 you can see the inverse relationship between volume and pressure.

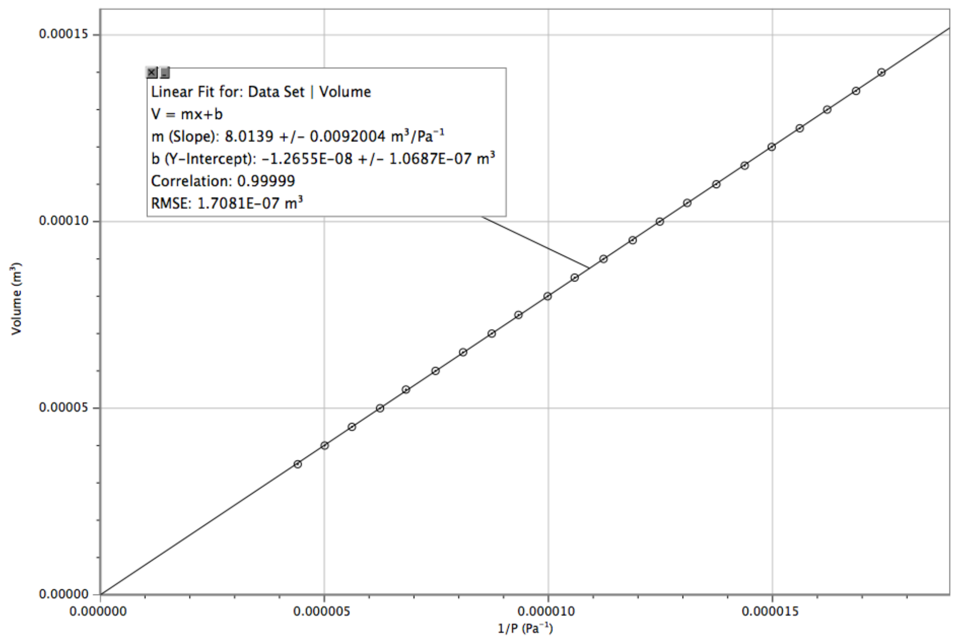
Comment [A12]: Communication. The data presentation, the graphs, and the text all make for easy understanding of what the student is doing.

Graph 1, *Volume against Average Pressure*



Boyle's law states that $P_1V_1 = P_2V_2 = P_nV_n$ which is to say that as the volume gets smaller the pressure gets larger. If $PV = \text{constant}$ then volume is inversely proportional to the pressure, that is, $V \propto \frac{1}{P}$. This is illustrated in Graph 2.

Graph 2, *Volume against Reciprocal of Average Pressure*



The best-fit line is clearly linear and, accounting for uncertainty in the gradient, includes the origin, hence the relationship is proportional (within experimental error). The most significant uncertainty is in the volume measurement, and when a graph including error bars of ±1 cc is added, the bars were too small to see so I did not include them here. Instead, I will use the statistical uncertainty generated by the computer, namely a gradient of 8.0139 ±0.0092004 with a negative volume intercept of –1.2655E–8 ± 1.068E–7. These values simplifies as follows:

$$\text{slope} = \left(8.014 \pm 0.009\right)\text{m}^3\text{Pa}$$

$$y - \text{axis intercept} = -1.26 \times 10^{-8}\,\text{m}^3$$

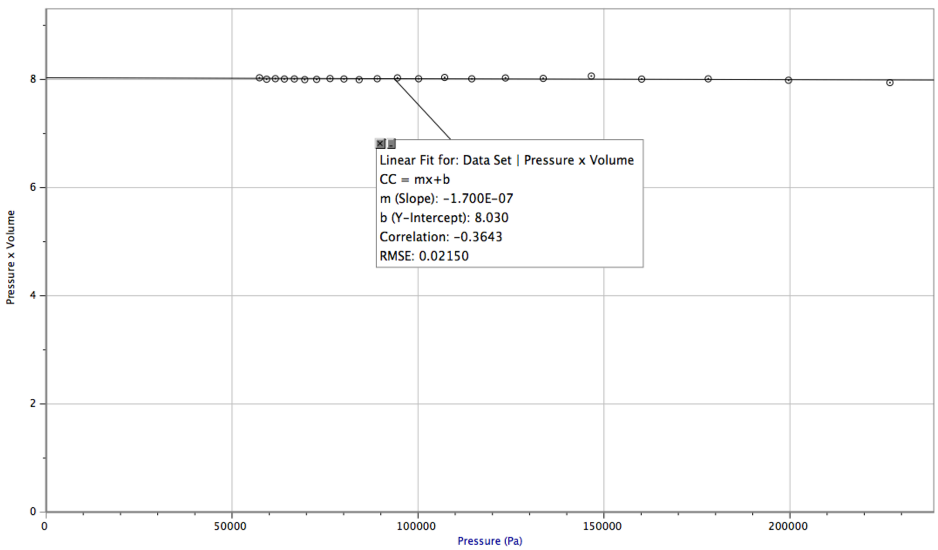
The gradient thus has an uncertainty of about 0.1%. The *y*-intercept is as close to zero as one would expect. The uncertainty here should not be expressed as a percentage as the ideal value would indeed be zero. If the *y*-intercept is in the order of 10^{-8} and the values of volume are in the order of 10^{-4} and 10^{-5} then we can appreciate any systematic shift as insignificant, probably due to rounding errors.

POSSIBLE DEAD SPACE IN VOLUME MEASUREMENTS

Comment [A13]: **Analysis**. The student is insightful here, both to account for the dead space and the manner in which they determine the dead space. A graph of PV against V was on an IB exam paper.

The small value of the y -intercept on Graph 2 could represent the dead space in the syringe and pressure sensor, but it was so small I ignored it. However, I want to double-check my values here. To do this I graphed the product of average pressure and volume against volume. A small and negative gradient represents the dead space[9].

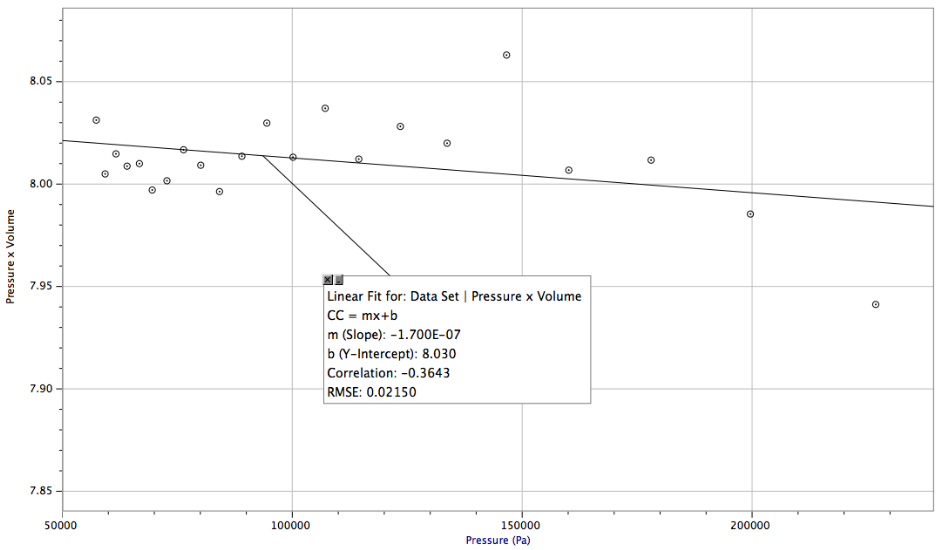
Graph 3, *Average Pressure x Volume against Average Pressure*



The slope is a $-1.7\times10^{-7}\,\mathrm{m}^3$. This is much larger than the offset represented in Graph 2, which was $-1.3\times10^{-8}\,\mathrm{m}^3$ or about 13 times larger. In either case, the negative slope is extremely small. The difference may be due to rounding errors in the calculations. And in either case it is much smaller than the ±1 cc uncertainty of any given measurement. I will therefore not assume this value represents the dead-space. However, both Graphs 2 and 3 suggest a possible negative value of volume that indeed represents the dead space and should be added to all volume measurements.

Graph 4 is a close up view of the scatter of data. Although this may look almost random, if you consider the scale we can say there is ever so slight a negative slope (representing dead space).

Graph 4, *A Close Up View of the Average Pressure x Volume against Average Pressure*



CALCULATING THE UNIVERSAL GAS CONSTANT

Comment [A14]: **Evaluaiton**. The student answers the research question in magnificent detail, with full appreciations of the strength and limitations of the experimental results.

First I found the number (or fraction there of) of moles in the air sample. Using a sensitive digital balance from the chemistry department I measured the syringe with 140 cc of air in it and again without air in it. The difference is the mass of air that I used in my experiment.

$$m_{\text{air}} = m_{\text{syringe with air}} - m_{\text{syringe without air}} = (73.323 - 73.225)\,\text{g} = 0.098\,\text{g}$$

Next I found the molar mass of air. Air is a gas made of up different percentages of various gases, such as N_2, O_2, Ar, and CO_2. I used an Internet calculator to determine the molar mass and was given as an effective average of 28.97 g/mole. This was confirmed by another web site.[10]

The number of moles of air in my syringe is thus calculated as:

$$n = \frac{m_{\text{air}}}{\text{molar mass of air}} = \frac{0.098\,\text{g}}{28.97\,\text{g/mol}} = 3.38281 \times 10^{-3}\,\text{mole}$$

From Graph 2 we find that the Boyle's law constant (for the given mass of air and constant temperature) from the gradient of the graph, *PV*, is equal to 8.0139 m³ Pa.

The temperature was $T = 21.0°\text{C} = (21.0 + 273.15)\,\text{K} = 294.15\,\text{K} \approx 294.2\,\text{K}$

Now for **the final result of my exploration**, the experimental value of the universal ideal gas law constant. My result is rounded to three significant figures.

$$R_{\text{experiment}} = \frac{PV}{nT} = \frac{\text{gradient}}{nT}$$

$$R_{\text{experiment}} = \frac{8.0139\,\text{m}^2\text{Pa}}{\left(3.38281\times10^{-3}\,\text{moles}\right)\left(294.15\,\text{K}\right)} = 8.05373\ \text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$$

$$\therefore R_{\text{experiment}} \approx 8.05\ \text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$$

Compare this with the textbook or accepted value of:

Comment [A15]: **Evaluaiton**. The student compares their experimental result with the accepted value, and appreciate the uncertainty range in both value.

$$R_{\text{accepted}} = \left(8.3144621 \pm 0.0000075\right)\ \text{J mol}^{-1}\ \text{K}^{-1} \approx 8.31\ \text{J mol}^{-1}\ \text{K}^{-1}$$

Compared to the accepted value, my result is low by about 3%.

CONCLUSION AND IMPROVEMENTS

My experimental value was about 3% off the accepted value, and on the low side. However, when I calculate 3% of my answer I get an uncertainty of ±0.24 J mole^{-1} K^{-1}, and this is in the first decimal place, the tenths. Perhaps my conclusion should be

$$R_{\text{exp}} = \left(8.1 \pm 0.2\right)\ \text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$$

With this expression my result almost reaches the accepted value. Recall that the uncertainty in the volume ranged from 0.2% to 2% and the uncertainty in the slops was only 0.1% while the two significant figures in the air mass determination represents an uncertainty of 1%. These are all very small, and my best determination of the quality of my results can be based on comparison to the accepted value, again at about 3%.

My purpose in this exploration was to establish the universal gas law constant by experiment. Overall, I was really excited when my conclusion confirmed to a reasonable degree the accepted value. I can hardly believe that this was possible.

IMPROVEMENTS TO THE EXPERIMENT

Comment [A16]: **Evaluation**. The student clearly discusses improvement that are relevant to their work. The student even mentions an interesting extension (about determining the atmospheric pressure).

(1) My biggest uncertainty is in the measurement of the number of moles. It is this area that would need improvement. Two significant figures in the mass measurement limited the mole number precision. My teacher said that were more sensitive balance scales are available but my school did not have one.

(2) My raw data did not account for the sensor's dead space. Although my two calculations are significantly different for the dead space in the sensor, I still believe that there is dead space and this volume should be added to each raw data value of volume. If I could account for this before performing all investigation my results would be more accurate. *Vernier's* technical specification says that no calibration is needed but they do not mention dead space. A personal email from *Vernier's* Jake Hopkins at www.vernier/til/582/ stated the dead space should be 0.8 mL (that is 0.8 cc). My uncertainty for volume was 1 cc. perhaps there is an unknown systematic error in all my data that somehow cancels out the dead space.

To account for the dead space (if I were to repeat my experiment) I could (a) open the sensor valve so that air goes in and remains at room pressure. (b) Then I could move the piston of the syringe down so the volume reading is zero. (c) Next I would close the valve and connect the syringe to the sensor, and then move the piston on the syringe so that the volume reads zero. The reading should be close to one atmosphere, but I would note the exact pressure reading. (d) Finally, I would pull the piston out slowly, noting the pressure reading. When the reading drops to half of its original reading, I would have double the volume. Hence the new piston reading is the volume of the pressure sensor and the tube up to the syringe. Unfortunately, I cannot spend more time on this project.

Comment [A17]: Evaluation. The student offers a most impressive appreciation of the dead space.

(3) With more time I would like to repeat the investigation using a larger syringe and perhaps with air at different temperature (as well as different gases).

(4) An alternative investigation that fascinates me would be to seal a syringe with the plunger 90% of the way in at normal atmospheric air pressure. The I would hand masses (converted to force units of weight) and record the handing weight as a function of syringe's partial vacuum volume. By measuring the diameter of the plunger and extrapolating form an appropriate graph the normal air pressure in the room (in units of N m^{-2}) can be determined. I would love to do this with more time.

Comment [A18]: Personal Engagement. Another expression of the student's enthusiasm.

FOOTNOTES

1 http://en.wikipedia.org/wiki/Gas_constant

2 http://www.chm.davidson.edu/vce/gaslaws/gasconstant.html

3 http://www.newworldencyclopedia.org/entry/Joseph_Louis_Gay-Lussac and http://www.chm.davidson.edu/vce/gaslaws/

4 Sybil and Parker, *McGraw-Hill Encyclopedia of Science and Technology*, McGraw-Hill Publishers, 1997, page 158.

5 "Boltzmann Constant." *Encyclopedia Britannica*, 2010. Encyclopedia Britannica Online. 10 Dec. 2010 in semi-conductor physics.

6 Tipler, Paul Allen, "Physics for Scientists and Engineers," Volume 1, Page 632.

7 http://www.vernier.com/products/sensors/gps-bta/

8 http://www.mathsisfun.com/data/standard-deviation-formulas.html

9 See "More on Systematic Error in a Boyle's Law Experiment" by Richard McCall in *The Physics Teacher*, Volume 50, Number 1, January 2012, pages 22-23.

10 The two web sites that derived the molar mass of air were:
http://www.engineeringtoolbox.com/molecular-mass-air-d_679.html
http://www.wolframalpha.com/input/?i=air+molecular+weight

MODERATOR'S COMMENTS

File Name: Physics_IA_SSS_11_Gas_Constant.docx

IA Type: Hands On Experiment for Subject Specific Seminar use only (not TSM sample)

Lab Report Title: "Determining the General Gas Law Constant, *R*"

MODERATOR COMMENTS: Total Marks = 24/24

Personal Engagement: 2/2
Although there are only a few expressions of personal engagement, the excitement the genuine interest is clearly expressed. Moreover, when we appreciate the subtle details demonstrated throughout this investigation it is clear that the student has the curiosity, intellectual interest and determined creativity in developing a most thorough scientific study. The moderator is certain that the student earns full marks for Personal Engagement, level 2.

Exploration: 6/6
The student states a clear and focused research question and sets the general gas law constant in appropriate scientific context. The techniques and methods are most appropriate for high school lab work. The student is aware of all the major and subtle issues involved. Although no safety issues or environmental concerns are mentioned, we can consider the fourth bulleted descriptor as an outlier for this investigation. The moderator is happy to award the student full marks here, level 6.

Analysis: 6/6
There is no doubt that the student has selected, recorded, processed and interpreted the data in a most appropriate and revealing way. Attention to detail, errors and uncertainties, is exceptional. There is more than enough data, limited only by the size of the syringe, and the precision is amazing and the accuracy is good. There is full awareness of the impact of uncertainties. The conclusion, a numerical value for *R*, is easily established and compared with the accepted value. The moderator is happy to award the student full marks for Analysis, 6.

Evaluation: 6/6
The student's report provides overwhelming evidence of the evaluation of the investigation and the results within the accepted scientific context. The conclusions (each required measurement) are described and justified, and in far more detail than one normally expects at high school level. Strengths and weakness of the method and data, and the sources of uncertainties are discussed in detail. The methodology is fully understood. The improvements and an extension are relevant and interesting. The moderator is happy once again to award the student full marks, 6.

Communication: 4/4
The entire report demonstrates effective communications; the method stays focused and the logical progression of the investigation follows the presentation. The report is concise despite containing many details. The terminology and scientific conventions are all followed. Overall, this is a most impressive investigation, and the moderator once again awards full marks, 4.